Algebraic Topology SS19
Exercise set 7.

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Problem 1. (left from the last time) Suppose X is a path connected, locally connected and locally simply connected space, let $x_0 \in X$. Let $\rho : \tilde{X} \to X$ be a covering. The automorphism group $\mathrm{Aut}(\rho)$ is the group of homeomorphisms $g : \tilde{X} \to \tilde{X}$ satisfying $\rho \circ g = \rho$. We call a covering ρ *normal* if $\mathrm{Aut}(\rho)$ acts transitively on $\rho^{-1}(x_0)$. Show that connected normal coverings of X are in bijection with normal subgroups of $\pi_1(X, x_0)$. Show that for a normal covering corresponding to a normal subgroup $H \subset \pi_1(X, x_0)$, the automorphism group is isomorphic to the quotient $\pi_1(X, x_0)/H$ (Hint: use functoriality from the lecture). A connected covering $\rho' : X' \to X$ is called a subcovering of $\rho : \tilde{X} \to X$ if there exists a map $f : \tilde{X} \to X'$ such that $\rho = \rho' \circ f$. Show that subcoverings of a normal covering $\rho : \tilde{X} \to X$ are in bijection with subgroups of $\mathrm{Aut}(\rho)$ (main theorem of Galois theory).

Problem 2. Compute homology of the Klein bottle using the triangulation presented at the lecture.

Problem 3. Compute homology of the sphere using the triangulation with 4 triangles.

Problem 4. [Hatcher, p. 131, Ex. 1] What familiar space is the quotient of a 2-simplex $[v_0, v_1, v_2]$ by the equivalence relation identifying $[v_0, v_1]$ and $[v_1, v_2]$, preserving the ordering of vertices. Compute its homology.

Problem 5. [Hatcher, p. 131, Ex. 7] Find a way of identifying pairs of faces of Δ^3 to produce a simplicial complex structure on S^3 having a single 3-simplex, and compute the simplicial homology groups of this simplicial complex.

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